A Mathematical Model For MIMO Imaging

Yufeng Cao[1,a], Juan F. Lopez Jr.[2,a], Alex Martinez[3,a], Zhijun Qiao[4,a]

[a]Department of Mathematics, The University of Texas-Pan American, Edinburg, TX 78539 USA

ABSTRACT

Multiple Input Multiple Output- MIMO Radar is a fast growing research area. This paper will give a brief introduction to the subject as well as derive an image formation scheme. The general problem of radar imaging is to use some physical model for a transmitted signal, and measurements of the signal that is scattered back to a receiver by a scene to attempt to derive information about the scene. The concept of communication involves a message sender, a message receiver, and a channel. The sender sends a message through the channel to the receiver. The receiver attempts to recover the original message. MIMO communication is just communication that involves sending several messages to several recipients. The problem of Multiple Input Multiple Output Radar Imaging is to use the corruption of transmitted messages to try and derive useful information about the environment that the messages traveled through. The extra information gained with MIMO Radar can be used to get rid of false targets, detect moving targets, and create a better resolution image. The plan for this research is to culminate to an in-scene 3-d Image reconstruction algorithm. The model presented provides a context in which to examine this problem.

Keywords: MIMO RADAR, RADAR imaging, wave equation, discrete Fourier transform, Born approximation, far field approximation, channel matrix, under determined linear systems

1. INTRODUCTION

MIMO RADAR systems utilize multiple transmit and receive antennas. This multiarray setup is advantagous because it can improve target detection and parameter identifability. However MIMO radar is a relatively new field with many open problems. As such, the field lends itself to mathematical study. This paper develops a mathematical model for MIMO Radar, and uses it to work out an Image formation algorithm. In section two we present the mathematical background necessary to derive the model. Section three uses the scalar wave model to describe the propagation of EM(Electromagnetic) waves based on Maxwell's equations.[1,2] Section four describes a MIMO configuration with isotropic(equally radiating in all directions) antennas.[3] Section five analyzes a channel matrix model relating the input and output signals.[4] In the sixth section we discuss sampling. In section seven we develop an image recovery algorithm. In section eight we present simulated data and test our image recovery algorithm. The conclusion summarizes our findings and discusses possible further research.

2. BACKGROUND

2.1 Fourier Transform

For $f : \mathbf{R}^d \to \mathbf{C}$, that is f maps a d-dimensional real vector to a complex number, let F denote the **Fourier transform**[5] of f defined as

$$F(\xi) = \int_{\mathbf{R}^d} f(x) e^{-2\pi i x\cdot\xi}\, \mathrm{d}x. \tag{1}$$

Further author information: (Send correspondence to)
1: Ycao1@broncs.utpa.edu
2: jflopezjr@utexas.ed
3: mnight08@yahoo.com
4: qiao@utpa.edu

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Similarly, if $u : \mathbf{R}^3 \times \mathbf{R} \to \mathbf{C}$ as $(x,t) \mapsto u(x,t)$ we define the **temporal Fourier transform** $U(x,f)$ transforming $t \to f$ as

$$U(x,f) = \int_{\mathbf{R}} u(x,t) e^{-2\pi i t f}\ \mathrm{d}t.$$

We will refer to both the full transform and one variable transform as the Fourier transform when there is no ambiguity. Henceforth, capital letters will denote Fourier transforms, i.e. F will denote the Fourier transform of the function f. We abbreviate Fourier Transform as FT when necessary.

2.2 Discrete Fourier Transform

For a discrete function of d variables $f : N_1 \times ... \times N_d \to \mathbf{C}$ where $N_i = \{0, ...M_i - 1\}$ define the **Discrete Fourier transform**[5] of f denoted $F(k_1, k_2..., k_d)$ as

$$F(k_1, k_2..., k_d) = \sum_{n_1=0}^{M_1-1} ... \sum_{n_d=0}^{M_d-1} f(n_1, n_2..., n_d) e^{-2\pi i (\frac{k_1 n_1}{M_1} + ... + \frac{k_d n_d}{M_d})}. \tag{2}$$

The **Inverse Discrete Fourier Transform**[5] of $F : K_1 \times ... \times K_d$ is given by

$$f(n_1, n_2, ..., n_d) = \frac{1}{M_1 M_2 ... M_d} \sum_{k_1=0}^{M_1-1} ... \sum_{k_d=0}^{M_d-1} F(k_1, k_2..., k_d) e^{2\pi i (\frac{k_1 n_1}{M_1} + ... + \frac{k_d n_d}{M_d})}. \tag{3}$$

We abbreviate the Discrete Fourier Transform and Inverse Discrete Fourier Transform as DFT, and IDFT respectively.

2.3 Periodicity of DFT

Let $F(k_1, k_2..., k_d)$ denote the DFT of $f : N_1 \times ... \times N_d \to \mathbf{C}$. F is periodic in each of its arguments[5] and satisfies

$$F(k_1 + M_1, k_2 + M_2..., k_d + M_d) = F(k_1, k_2..., k_d). \tag{4}$$

2.4 Realness Property of FT and DFT

Let $f : \mathbf{R}^d \to \mathbf{R}$ then the Fourier transform of f satisfies[5]

$$F(-\xi) = \overline{F(\xi)} \tag{5}$$

Similiarily if $f : N_1 \times ... \times N_d \to \mathbf{R}$ the DFT of f satisfies

$$F(-k_1, -k_2..., -k_d) = \overline{F(k_1, k_2..., k_d)} \tag{6}$$

These can be verified by simply dsitributing the conjugation to the integral or sum and factoring out the negative in the exponent.

2.5 Dirac Delta Function

The Dirac delta function is the function that satsifies

$$\delta(x) = \{0, x \neq 0\} \tag{7}$$

and

$$\int \delta(z) dz = 1. \tag{8}$$

This function is zero everywhere except for when its argument is zero. We use this fucntion to establish the notion of a point isotropic object. It can be shown that for a smooth function $f : \mathbf{R}^d \to \mathbf{C}$ that

$$\int f(z) \delta(z - x)\ \mathrm{d}z = f(x). \tag{9}$$

2.6 Convolution

The convolution operator in space and time $*_{x,t}$ of $f : \mathbf{R}^d \times \mathbf{R} \to \mathbf{C}$ and $g : \mathbf{R}^d \times \mathbf{R} \to \mathbf{C}$ is given by

$$f(x,t) *_{x,t} g(x,t) = \int_{\mathbf{R}^d} \int_{\mathbf{R}} f(x-z, t-\tau) g(z,\tau) d\tau \, \mathrm{d}z \tag{10}$$

Similarily the convolution only in space $*_x$ is given by

$$f(x,t) *_x g(x,t) = \int_{\mathbf{R}^d} f(x-z,t) g(z,t) \, \mathrm{d}z. \tag{11}$$

The Convolution Theorem relating convolution to multiplication when transforming in time is given by

$$\int_{\mathbf{R}} f(x,t) *_{x,t} g(x,t) e^{-2\pi i t f} \, \mathrm{d}t = F(x,f) *_x G(x,f) \tag{12}$$

2.7 The Wave Equation

The wave equation for $u : \mathbf{R}^d \times \mathbf{R} \to \mathbf{C}$ in free space is given by

$$\left(\nabla^2 - c_0^{-2} \partial_t^2\right) u(x,t) = -j(x,t). \tag{13}$$

The Greeen's function $g_0(x,t)$ for (13) given by

$$g_0(x,t) = \frac{\delta(t - \frac{|x|}{c_0})}{4\pi|x|} \tag{14}$$

can be used to solve (13) as

$$u(x,t) = \int_{\mathbf{R}^3} \int_{\mathbf{R}} g_0(x-z, t-\tau) j(z,\tau) d\tau dz, \tag{15}$$

2.8 The Far Field Approximation

Suppose we are given two points $x, y \in \mathbf{R}^d$. We wish to approximate $|x-y|$ and $\frac{1}{|x-y|}$.[1] To do this we first note that

$$|x-y| = \sqrt{(x-y)\cdot(x-y)} = \sqrt{|x|^2 - 2x\cdot y + |y|^2} = |x|\sqrt{1 - 2\frac{\widehat{x}\cdot y}{|x|} + \frac{|y|}{|x|}^2} \tag{16}$$

If we use the binomial series we can find that for $x \ll 1$

$$\sqrt{1-x} \approx 1 - \frac{x}{2} \tag{17}$$

and that

$$\frac{1}{\sqrt{1-x}} \approx 1 + \frac{x}{2}. \tag{18}$$

This means that if $|x| \gg |y|$ then (16) becomes

$$|x-y| \approx |x| - \widehat{x}\cdot y \tag{19}$$

and

$$\frac{1}{|x-y|} = \frac{1}{|x|\sqrt{1 - 2\frac{\widehat{x}\cdot y}{|x|} + \frac{|y|}{|x|}^2}} \approx \frac{1}{|x|} + \frac{\widehat{x}\cdot y}{|x|^2} \approx \frac{1}{|x|} \tag{20}$$

3. SCALAR WAVE MODEL

We adopt the following wave equation model for the propagation of the scalar electromagnetic wave[1,2] $u(x,t)$ for $x \in \mathbf{R}^3$ and $t \in \mathbf{R}$ with source $j(x,t)$:

$$\left(\nabla^2 - c^{-2}(x)\partial_t^2\right) u(x,t) = -j(x,t). \tag{21}$$

we adopt the principle that the total field at any position in space x and any time t is the superposition of incident and scattered fields we write

$$u(x,t) = u_{\text{in}}(x,t) + u_{\text{sc}}(x,t). \tag{22}$$

The incident field is the field in the abscense of scatterers and satisfies

$$\left(\nabla^2 - c_0^{-2}\partial_t^2\right) u_{\text{in}}(x,t) = -j(x,t). \tag{23}$$

Define the **continous reflectivity** function $v_{continous}(x)$ as

$$v_{continous}(x) = \frac{1}{c_0^2} - \frac{1}{c^2(x)} \tag{24}$$

Our goal is to find some approximation of $v_{continous}(x)$ as it gives us an estimation of target distribution. In this model we have ignored moving targets as $v_{continous}(x)$ does not depend on time. Under the weak scaterring assumption (the Born approximation) $u_{\text{in}} \approx u$. By subtracting (21) from (23) we find that u_{sc} satisfies

$$\left(\nabla^2 - c_0^{-2}\partial_t^2\right) u_{\text{sc}}(x,t) \approx -v_{continous}(x)\partial_t^2 u_{\text{in}}(x,t). \tag{25}$$

An approximation to the scattered field is then given by

$$u_{\text{sc}}(x,t) \approx g_0(x,t) *_{x,t} v_{continous}(x)\partial_t^2 u_{\text{in}}(x,t). \tag{26}$$

Applying the Fourier transform taking t to f, and lettting $\omega = 2\pi f$, the transformed fields satisfy

$$\left(\nabla^2 + c^{-2}(x)\omega^2\right) U(x,f) = -J(x,f) \tag{27}$$

$$\left(\nabla^2 + c_0^{-2}\omega^2\right) U_{\text{in}}(x,f) = -J(x,f) \tag{28}$$

where J is the Fourier transform of j. The Green's function $g_0(x,t)$ becomes $G_0(x,f)$

$$G_0(x,f) = \frac{e^{2\pi i f|x|/c_0}}{4\pi|x|} \tag{29}$$

Using the convolution theorem (26) becomes

$$U_{\text{sc}}(x,f) = -G_0(x,f) *_x w^2 v_{continous}(x) U_{\text{in}}(x,f) = -\int_{\mathbf{R}^3} \frac{e^{2\pi i f|x-z|/c_0}}{4\pi|x-z|} v_{continous}(z)\omega^2 U_{\text{in}}(z,f)\,\mathrm{d}z. \tag{30}$$

4. MIMO DATA MODEL

Henceforth we shall work in the transformed frequency domain, i.e. with f and ω rather than t. In a MIMO RADAR system we have N_T different transmitting antennas, and N_R recieveing antennas. Each transmitter generates a signal $p_\ell(t)$ which has Fourier Transform $P_\ell(f)$. Let x_ℓ and y_j denote the positions in 3-d space of the ℓth trasmmitter and jth receiver respectively. Adpoting a pointlike antenna model we can model the source term $J(x,f)$ as a super position of the individual transmitter signals located at their respective locations. That is $J(x,f)$ can be expressed as

$$J(x,f) = \sum_{\ell=1}^{N_T} P_\ell(f)\delta(x - x_\ell). \tag{31}$$

We obtain the incident field by solving (28) as

$$U_{\rm in}(x,f) = \sum_{\ell=1}^{N_T} \int_{\mathbf{R}^3} \frac{e^{i\omega|x-y|/c_0}}{4\pi|x-y|} P_\ell(f)\delta(y - x_\ell)\,\mathrm{d}y \tag{32}$$

so that

$$U_{\rm in}(x,f) = \sum_{\ell=1}^{N_T} P_\ell(f) \frac{e^{i\omega|x-x_\ell|/c_0}}{4\pi|x-x_\ell|}. \tag{33}$$

We can substitute (33) into (30) and solve for the scattered field measured at the jth receiver as

$$U_{\rm sc}(y_j,f) = -\sum_{\ell=1}^{N_T} \omega^2 P_\ell(f) \int_{\mathbf{R}^3} \frac{e^{i\omega(|y_j-z|+|z-x_\ell|)/c_0}}{(4\pi)^2|y_j-z||z-x_\ell|} v_{continous}(z)\,\mathrm{d}z. \tag{34}$$

We define the **receive vector** $R(f)$ as

$$R(f) = \begin{pmatrix} U_{\rm sc}(y_1,f) \\ \vdots \\ U_{\rm sc}(y_{N_R},f) \end{pmatrix}. \tag{35}$$

The receive vector contains all the data measured at the receivers for fixed frequency sample f.

5. CHANNEL MATRIX

5.1 Matrix Form

From (34), define the matrix $H(f) \in \mathbf{C}^{N_R \times N_T}$ with j,ℓth entry $H_{j,\ell}(f)$

$$H_{j,\ell}(f) = -\int_{\mathbf{R}^3} \omega^2 \frac{e^{i\omega(|y_j-z|+|z-x_\ell|)/c_0}}{(4\pi)^2|y_j-z||z-x_\ell|} v_{continous}(z)\,\mathrm{d}z \tag{36}$$

and the transmit vector P as

$$P = \begin{pmatrix} P_1(f) \\ \vdots \\ P_{N_T}(f) \end{pmatrix}. \tag{37}$$

Then

$$R(f) = H(f)P(f). \tag{38}$$

H describes the channel between each antenna receiver pair. Because of this we call H the **channel matrix**.

5.2 Approximations

Assuming that $|x_\ell| \gg |z|, |y_j| \gg |z|$ for all $z \in \mathrm{supp}(V)$ we apply the far-field approximation to find

$$|x_\ell - z| \approx |x_\ell| - \widehat{x}_\ell \cdot z \tag{39}$$

$$|y_j - z| \approx |y_j| - \widehat{y}_j \cdot z \tag{40}$$

and that

$$\frac{1}{|x_\ell - z|} \approx \frac{1}{|x_\ell|} \tag{41}$$

$$\frac{1}{|y_j - z|} \approx \frac{1}{|y_j|}. \tag{42}$$

Substituting these into (36) we find that

$$H_{j,\ell}(f) \approx -\frac{\omega^2 e^{i\omega(|y_j|+|x_\ell|)}}{(4\pi)^2 |y_j||x_\ell|} \int_{\mathbf{R}^3} e^{-i\omega z\cdot(\widehat{y}_j + \widehat{x}_\ell)/c_0} v_{continous}(z)\, \mathrm{d}z \tag{43}$$

so (43) becomes

$$H_{j,\ell}(f) \approx -\frac{\omega^2 e^{i\omega(|y_j|+|x_\ell|)}}{(4\pi)^2 |y_j||x_\ell|} V_{continous}\left(\frac{f}{c_0}\left(\widehat{y}_j + \widehat{x}_\ell\right)\right) \tag{44}$$

where $V_{continous}$ denotes the spatial Fourier transform of $v_{continous}$. In the remainder of this paper we develop and test an imaging scheme based on this relation.

6. SAMPLING

The point of the entire paper is to form an image approximating v. If we know $H(f)$ for all f, then we approximately know $V_{continous}$ along the $N_T N_R$ lines $\frac{f}{c_0}\left(\widehat{y}_j + \widehat{x}_\ell\right)$ in the Fourier domain by the relation

$$V_{continous}\left(\frac{f}{c_0}\left(\widehat{y}_j + \widehat{x}_\ell\right)\right) \approx H_{j,\ell}(f) \frac{(4\pi)^2 |y_j||x_\ell| e^{-i\omega(|y_j|+|x_\ell|)}}{\omega^2} \tag{45}$$

when $f \neq 0$.

The first step away from theory and towards proccessing is establishing a useful sampling scheme that relates our sampled recieve vector to the sampled channel matrix and transmit vector. We collect samples of $R(w)$ and $P(w)$ over the set of sampled frequencies $\{f_1, \ldots, f_{N_S}\}$.

We define our sampled receive vector as

$$\mathcal{R} = \begin{pmatrix} R(f_1) \\ \vdots \\ R(f_{N_S}) \end{pmatrix}. \tag{46}$$

and our sampled transmit vector as

$$\mathcal{P} = \begin{pmatrix} P(f_1) \\ \vdots \\ P(f_{N_S}) \end{pmatrix}. \tag{47}$$

We then define our sampled channel matrix as

$$\mathcal{H} = \begin{pmatrix} H(f_1) & \cdots & (0) & & (0) \\ \vdots & \ddots & & & \\ (0) & & H(f_n) & \cdots & (0) \\ \vdots & & \vdots & \ddots & \\ (0) & & (0) & & H(f_{N_S}) \end{pmatrix}. \tag{48}$$

where (0) is the $N_R \times N_T$ zero matrix. $\mathcal{H}$ is the $N_R N_S \times N_T N_S$ matrix containing only zeros except the block diagonal which is H sampled at different frequencies.

These definitions give rise to the equation

$$\mathcal{R} = \mathcal{H}\mathcal{P} \tag{49}$$

We must then estimate $\mathcal{H}$. There are several methods for estimatting the channel depending on whether we have prior information about the distribution of the measurment error or the channel itself. In this paper we do not focus on estimating the channel matrix.

7. IMAGE FORMATION

Once we have estimated our sampled channel matrix we can move towards performing image recovery. For the sake of simplicity we focus on recovering a 2-d image, although it extends easily to 3-d. We restict our scene to consist of a finite number of point like scatterers that lie on a grid of $M_1 \times M_2$ points on the xy plane. The x_1 and x_2 spacing of the points are given by Δx_1 and Δx_2 respectively. Complicated targets can be approximated by a sufficiently dense grid. $v_{continous}$ is then a function of just two variables x_1 and x_2. We denote the (n_1, n_2)th position on the grid as $(x_{1_{n_1}}, x_{2_{n_2}})$. This point can be found explicitly as

$$(x_{1_{n_1}}, x_{2_{n_2}}) = (n_1 \Delta x_1, n_2 \Delta x_2). \tag{50}$$

The discrete reflectivity function $v_{discrete}(n_1, n_2)$ is defined as

$$v_{discrete}(n_1, n_2) = v_{continous}(x_{1_{n_1}}, x_{2_{n_2}}). \tag{51}$$

The fact that the scatteres lie on a grid means $v_{continous}$ must be of the form

$$v_{continous}(x_1, x_2) = \sum_{n_1=0}^{M_1-1} \sum_{n_2=0}^{M_2-1} v_{discrete}(n_1, n_2) \delta(x_1 - x_{1_{n_1}}) \delta(x_2 - x_{2_{n_2}}). \tag{52}$$

The 2-d FT of $v_{continous}$ then becomes

$$V_{continous}(\xi_1, \xi_2) = \sum_{n_1=0}^{M_1-1} \sum_{n_2=0}^{M_2-1} v_{discrete}(n_1, n_2) e^{-2\pi i (x_{1_{n_1}} \xi_1 + x_{2_{n_2}} \xi_2)}. \tag{53}$$

Substituting the explicit values in for $x_{1_{n_1}}$, and $x_{2_{n_2}}$ we have

$$V_{continous}(\xi_1, \xi_2) = \sum_{n_1=0}^{M_1-1} \sum_{n_2=0}^{M_2-1} v_{discrete}(n_1, n_2) e^{-2\pi i (n_1 \Delta x_1 \xi_1 + n_2 \Delta x_2 \xi_2)}. \tag{54}$$

We define the discrete function $V_{discrete}(m_1, m_2)$ as

$$V_{discrete}(m_1, m_2) = V_{continous}\left(\frac{m_1}{M_1 \Delta x_1}, \frac{m_2}{M_2 \Delta x_2}\right) = \sum_{n_1=0}^{M_1-1} \sum_{n_2=0}^{M_2-1} v_{discrete}(n_1, n_2) e^{-2\pi i (\frac{n_1 m_1}{M_1} + \frac{n_2 m_2}{M_2})}. \tag{55}$$

$V_{discrete}$ is just the DFT of $v_{discrete}$ allowing us to simply use the IDFT to recover $v_{discrete}$ as

$$v_{discrete}(n_1, n_2) = \frac{1}{M_1 M_2} \sum_{m_1=0}^{M_1-1} \sum_{m_2=0}^{M_2-1} V_{discrete}(m_1, m_2) e^{2\pi i (\frac{n_1 m_1}{M_1} + \frac{n_2 m_2}{M_2})}. \tag{56}$$

If all of our scatters and antenna lie along the same line the equations reduce to

$$V_{discrete}(m_1) = V_{continous}\left(\frac{m_1}{M_1 \Delta x_1}\right) = \sum_{n_1=0}^{M_1-1} v_{discrete}(n_1) e^{-2\pi i (\frac{n_1 m_1}{M_1})}. \tag{57}$$

and

$$v_{discrete}(n_1) = \frac{1}{M_1} \sum_{m_1=0}^{M_1-1} V_{discrete}(m_1) e^{2\pi i (\frac{n_1 m_1}{M_1})}. \tag{58}$$

We can just as well evaluate $V_{discrete}$ at $(m_1 + k_1 M_1)$ in 1-d and $(m_1 + k_1 M_1, m_2 + k_2 M_2)$ in 2-d for integers k_1 and k_2 due to the periodicity of the DFT. This allows us to choose the points in fourier space that best suit the points our data can be used to estimate.

Our algorithm for image recovery can then be stated as follows:

1. Estimate sampled channel matrix.
2. Use the relation between the channel matrix and the FT of the reflectivity function to find $V_{continous}$ at several Points.
3. Use realness property of $v_{continous}$ to increase the number of points known of $V_{continous}$.
4. Interpolate the points of $V_{continous}$ and estimate its values at $(\frac{m_1}{M_1 \Delta x_1}, \frac{m_2}{M_2 \Delta x_2})$ to estimate $V_{discrete}$
5. Apply the 2-d IDFT to our estimate of $V_{discrete}$ to recover $v_{discrete}$

8. SIMULATION

Here we come to testing out the derived inversion scheme. We focus on recovering the image given the channel matrix. Our simulation is restricted to the 1-d case. Scatterers are placed along a line and used to generate several channel matrices according to (36).

First we place two scatterers at positions -100, and 100. They both have reflectivity -100. We test image recovery for 10, 100, 1000, 10000, 100000, and 1000000 frequency samples. The recovered images are below.

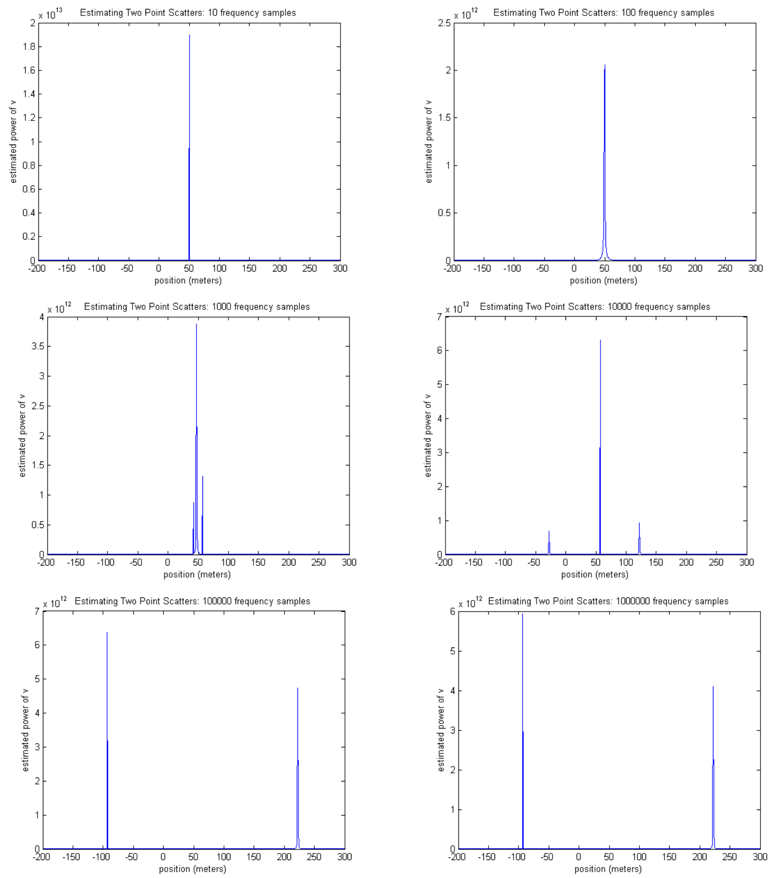

Figure 1. Imaging Two Point Scatterers

The distinction between two scatterers becomes much better as the number of frequency samples goes up. Now we try ten scatterers at different locations with equal reflectivity and again work with 10, 100, 1000, 10000, 100000, and 1000000 frequency samples. The locations of the scatterers are -100, -60, -20, -50, -9, -5, 0, 9, 100, and 300. Our recovered images are shown below.

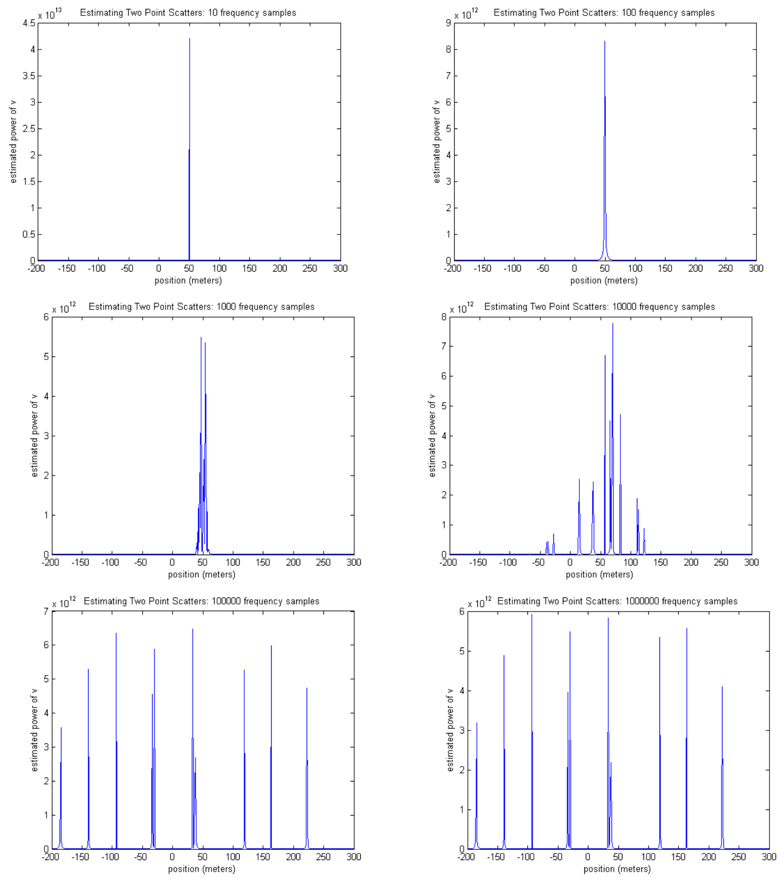

Figure 2. Imaging Ten Point Scatterers

When the number of samples is 1000000 all ten of the scatterers begin to become visible.

9. CONCLUSION

In this paper we have developed an image recovery algorithm for a MIMO RADAR configuration based on a wave equation model for the propagation of EM Waves. A convienient feature of this model is that it does not require slowly varying signals. We tested our approximations by recovering scatterers in 1-d. There are two major portions of image formation with this model. The first is estimating the channel matrix. The second is using the estimated fourier data to reconstruct the image. There is still much work to be done with this model such as estimating the channel matrix, finding optimal signals for image recovery, and moving towards a filtered backprojection algorithm. Noise and clutter are further additions that can be made to this model. There are two approximations we used in the derivation of this model. They are the Born approximation and the far field approximation. Finding ways to get around these approximations is subject to future research.

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