1 Polynomial Time

- All reasonable deterministic computational models are polynomially equivalent
- Focus on aspects of time complexity that are unaffected by polynomial differences in run-time.
- Develop a theory independent of any specific model
  - Look at fundamental properties of computation
- **Definition** (Sipser 286): P is the class of languages decidable in polynomial time on a deterministic single tape TM

\[ P = \bigcup_{k} \text{TIME}(n^k) \]  

P is invariant \forall models of computation that are polynomially equivalent. P roughly corresponds to problems that are particularly solvable.

\[ \text{PATH} = \{ <G, s, t>: G \text{ is a directed graph with a path from } s \text{ to } t \} \]

**Theorem:** Path \in P
The class NP

- Some problems seem to require exponential time

- **Definition** (Sipser 293): A verifier for a language A is an algorithm V, where
  \[ A = \{ w : v \text{ accepts } <w, c> \text{ for some string } c \} \]  
  - Time complexity based on |w|
  - C is the certificate, or proof of membership for A

HAMPATH \{ <G, s, t> : G is a directed graph with a path from s to t \}

- Path that visits each vertex once

- A certificate would be the sequence of vertices \( \pi_c \)

**Definition**: NP is the class of languages that have polynomial time vertices – *on final*

NP = non-deterministic polynomial time

**Theorem**: A language is the NP if and only if it is decided by a non-deterministic polynomial time TM

**Definition**: \( \text{NTIME}(t(n)) = \{ L : L \text{ is a language decided by an } O(t(n)) \text{ time non-deterministic TM} \} \)

\[ NP = \bigcup_{k=1}^{\infty} \text{NTIME}(n^k) \]
**Theorem:** CLIQUE ∈ NP

- C is the clique

V = "On input << G, k>, C>
1. Test whether C is a subgraph of G
2. Test whether G contains edges connecting vertices
3. If both pass, accept, otherwise we reject”

N = "On input < G , k >
1. Non-deterministic select a subset of C of size k in G
2. Test whether G contains edges connecting vertices
3. If yes, accept, else reject”

What about \(\overline{\text{CLIQUE}}\)?