Decidable – Halts when the string is and is not part of the language. Recognizable – Halts on all accepted inputs. It will halt on yes answers, but it may not halt on no answers.

**Multitape Turing Machines**

- Like an ordinary TM, but with multiple tapes
- Each tape has their own read/write head.
- Input is always on tape 1 and other tapes are blank.
- Transition function \( \delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k \)

\( k \) is the # of tapes

**Theorem:** Every Multitape Turing Machine has an equivalent single-tape Turing Machine

*Copy each tape into one tape. Separate each of the strings with a special symbol, and mark the heads*

\[ S = \text{an input } w = w_1...w_n \]

1. First S puts tape into format representing M tapes
   
   \[ # w_1w_2...w_n#_#_#_#_... \]

2. To simulate a single move, S scans the tape from the first # marking the leftmost end to the \((k+1)^k#\) (rightmost end) to determine the position of virtual heads. Then S makes a 2\text{nd} pass to update the tape according to M’s transition function.

3. If at any point S moves a virtual head onto a # (right), S marks spot with \( _\). Then moves everything over one spot until it reaches the end, then it continues simulation.

**Corollary:** A language is Turing recognizable if and only if some multiple Turing Machine recognizes it.
**Non-deterministic Turing Machine**

- Transition function $\delta: Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\})$
- Accepts if any branch of the computation accepts

**Theorem:** Every Non-deterministic Turing Machine has an equivalent deterministic Turing Machine.

**Proof Idea:**

- Simulate a Non-deterministic Turing Machine $N$ with a Deterministic Turing Machine $D$ by trying all branches of the computation.
- Can’t use Depth First Search (infinite loops).
- $D$ explores the non-deterministic computation tree breadth-first.
- Simulating $D$ has 3 tapes.
  1. Tape 1 has input string (read only)
  2. Tape 2 maintains a copy of $N$’s tape on some branch of its computation.
  3. Tape 3 keeps track of $D$’s location in $N$’s non-deterministic computation tree.

- Address all nodes by path taken to node from root $\Gamma_b = \{1, 2, 3, \ldots, b\}$
- $231$ – from root, 2nd child, then 3rd child and its 1st child.

**Corollary:** A language is Turing-recognizable if and only if some non-deterministic Turing Machine recognizes it.
Corollary: A language is decidable if and only if some non-deterministic Turing Machine describes it.