B = \{w#w: w \subseteq \{0,1\}^* \}

M_1 = (Q, \Sigma, \Gamma, S, q_{1}, q_{\text{acc}}, q_{\text{rej}})

Q = \{q_{1}, \ldots, q_{8}, q_{\text{acc}}, q_{\text{rej}} \}

\Sigma = \{0, 1, \#\} \text{ and } \Gamma = \{0, 1,\#, x, \_\}
A = \{ 0^{2n} : n \geq 0 \} 

M_2 = "On input string w:
  1. Sweep from left to right, crossing off every other 0
  2. If in stage 1, \( \exists \) a single 0, accept
  3. If in stage 1 the tape contained an odd # of 0s greater than 1, reject
  4. Return the head to the left-hand end of the tape
  5. Go to stage 1"

If we plug in string 0 0 0 0 0 0 0 0, the Turing machine steps will be:

\[
\begin{align*}
_ & \_ q_2 0 0 0 0 0 0 0 \\
_ & \_ x q_3 0 0 0 0 0 0 \\
_ & \_ x 0 q_4 0 0 0 0 0 \\
_ & \_ x 0 x q_5 0 0 0 0 \\
_ & \_ x 0 x 0 q_6 0 0 \\
_ & \_ x 0 x 0 x q_7 0 \\
_ & \_ x 0 x 0 x 0 q_8 x \\
_ & \_ q_5 x 0 x 0 x 0 x \\
_ & \_ q_5 x 0 x 0 x 0 x \\
_ & \_ q_2 x 0 x 0 x 0 x \\
\end{align*}
\]

\[
\begin{align*}
_ & \_ x q_2 0 x 0 x 0 x \\
_ & \_ x x q_4 x 0 x 0 x \\
_ & \_ x x x q_3 0 x 0 x \\
_ & \_ x x x 0 q_4 x 0 x \\
_ & \_ x x x 0 x x x q_3 \\
_ & \_ q_2 x x x 0 x x x \\
_ & \_ x x x x x x x q_3 \\
_ & \_ q_2 x x x x x x x \rightarrow \text{accept}
\end{align*}
\]
Def'n (S 170) A language is Turing recognizable if some Turing machine recognizes it.

- The collection of strings a Turing machine M accepts is the language of M or the language recognized by M.

Def'n: A language is Turing - decidable, or simply decidable, if some Turing machine decides it. *(Deciding means it halts on all inputs)*

Corollary: All Turing decidable languages are Turing recognizable, but not all recognizable languages are decidable.

$C = \{ a^i b^j c^k : i \times j = k \text{ and } i, j, k \geq 1 \}$

$M_3 = \text{"On input string w }$

1. Scan input from L to R and ensure string has form a*b*c', reject if not.
2. Return head to LHS
3. Cross off an a and scan to the right until a b occurs
   Shuffle between b's and c's crossing off one of each until all b's are gone
   If all c's are gone and b's remain, reject
4. Restore crossed off b's and repeat stage 3 if there is another a to cross off
   If all a's are crossed off, check if all c's are crossed off. If yes, accept; otherwise reject. "

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Ex: Elemental distinction problem - given a list of strings over \{0,1\} separated by #s, determine if all strings are unique \(E = \{ \#x_1\#x_2\#...\#x_i : \text{each } x_i \subseteq \{0,1\}^k \text{ and } x_i \neq x_j \text{ and } i \neq j \}\)

\(M_4 = \text{“On input } w:\)

1. Place mark on top of leftmost tape symbol. If symbol is blank, accept.
   If symbol is a #, continue w/ next stage. Otherwise, reject.
2. Scan right to next # and place mark on top. If no # is found before a blank symbol, only \(x_1\) exists, accept.
3. By zig- zagging, compare the two strings to the right of marked #’s. If equal, reject.
4. Move the rightmost of marked # to the next # symbol on right. If no # is encountered before a blank symbol, move the leftmost mark to the # after it. This time, if no # is available for rightmost # mark, all strings have been compared, so accept.
5. Go to stage 3. ”