In class quiz:
Give an NFA that accepts: $A = \{ w \text{ w has four zeros in a row or only one zero} \}$

$$E = \{ 0,1 \}$$
DFA:

Equivalent NFA:
Example NFA:
Thm: The class of regular languages is closed under union, concatenation, and * operation.

(Closed = under union, concatenation, and * operation you get something in regular language back)
Concatenation:
N1 accept to N2 start
N1 accept became non-accept for W

Thm (M51): The set of regular languages is closed under the complement and intersection operations.

1) \( \overline{A} = \{ w \in \Sigma^* : w \notin A \} \)
2) $A_1 \cap A_2 = \{ w \in A \text{ and } w \in A_2 \}$

Regular Expressions:

For the following examples: $\Sigma = \{0, 1\}$

- Ex (M52): $A = \{ w \mid w \text{ contains exactly 2 0's} \}$
  
  $$= 1^*01^*01^*$$

- Ex: $A = \{ w \mid w \text{ contains at least 2 0's} \}$
  
  $$= (1 \cup 0)^*0(1 \cup 0)^*0(1 \cup 0)^*$$

- Ex: $A = \{ w : |w| \mod 2 = 0 \}$ (length of $w$ is even)

  $$= ((1 \cup 0)(1 \cup 0))^*$$

Definition: $R$ is a regular expression if $R$ is:

1) $a$ for some $a \in \Sigma$
2) $\epsilon$
3) $\varnothing$ (empty)
4) $(R_1 \cap R_2)$ where $R_1$, $R_2$ are regular expressions
5) $(R_1R_2)$ where $R_1$, $R_2$ are regular expressions
6) $R_1^*$ where $R_1$ is a regular expression

- To distinguish between a regular expression $R$ the language it describes is $L(R)$.
- For convenience if $L(R_1) = L(R_2)$ we write $R_1 = R_2$
  
  - Ex: If $R_1 = (0 \cup \epsilon)1^*$ and $R_2 = 01^* \cup 1^*$ then $R_1 = R_2$
Given $R_1$, $R_2$, $R_3$ are regular expressions (regex), then:

1) $R_1 \varnothing = \varnothing R_1 = \varnothing$

2) $R_1 \varepsilon = \varepsilon R_1 = R_1$

3) $R_1 \cup \varnothing = \varnothing R_1 = R_1$

4) $R_1 \cup R_1 = R_1$

5) $R_1 \cup R_2 = R_2 \cup R_1$

6) $R_1 (R_2 \cup R_3) = R_1 R_2 \cup R_1 R_3$

7) $R_1 (R_2 R_3) = (R_1 R_2) R_3$

8) $\varnothing^* = \varepsilon$

9) $\varepsilon^* = \varepsilon$