CSCI 4325
Assignment 7 (100 points)

The homework is to be turned in by 5 P.M. on the day it is due. Style and correctness will be graded – be neat and thoroughly explain each step. Anything that is not clear will be counted wrong.

**Problem 1 (20):** Given a string \( w \in \{0, 1\}^* \), \( w^R \) is its reversal. Let \( T = \{<M> | M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\} \). Show that \( T \) is undecidable. (You can not use Rice’s Theorem before proving it)

**Problem 2 (20):** In the following instance of the Post Correspondence Problem, is there a match? Describe your approach to the problem. Knowing that the problem is undecidable, try to explain where your approach might fail.

\[
\begin{align*}
\begin{bmatrix} b \\ ab \end{bmatrix}, & \begin{bmatrix} a \\ ab \end{bmatrix}, \begin{bmatrix} aba \\ ba \end{bmatrix}, \begin{bmatrix} aba \\ a \end{bmatrix}
\end{align*}
\]

**Problem 3 (20):** Prove that \( EQ_{CFG} = \{<G_1, G_2> | G_1, G_2 \text{ are CFGs and } L(G_1) = L(G_2)\} \) is undecidable.

**Problem 4 (20):** Let RELPRIME = \( \{<x, y> | x \text{ and } y \text{ are positive integers that are relatively prime}\} \). Given the following algorithm to test if two positive integers are relatively prime, let \( n \) be the maximum number of decimal digits in \( x \) and \( y \). Analyze the running time of this algorithm, using \( O \)-notation. Explain the details and give your reasoning for each step.

On input \( <x, y> \) where \( x \) and \( y \) are positive integers.
1. Repeat until \( y = 0 \):
2. Assign \( x \leftarrow x \mod y \).
3. Swap \( x \) and \( y \).
4. Output \( x \). If the result is 1, accept; otherwise reject.

**Problem 5 (20):** Let \( S_1 \) be a countable set and \( S_2 \) be an uncountable set with \( S_1 \subseteq S_2 \). Prove that \( S_2 - S_1 \) is uncountable.

**Bonus (12):** Tetrominos are four squares stuck together along an edge (Tetris pieces). There are five distinct tetromino types: the straight, square, L-shaped, T-shaped, and Z-shaped tetromino. Is it possible to tile (i.e., cover exactly without overlaps) an 8 \times 8 chessboard with the following? If possible, give the tiling. If not, explain why.

(a) 16 straight tetrominoes
(b) 16 square tetrominoes
(c) 16 L-tetrominoes
(d) 16 T-tetrominoes
(e) 16 Z-tetrominoes
(f) 15 T-tetrominoes and one square tetromino