CSCI 4325
Assignment 6 (100 points)

The homework is to be turned in by 5 P.M. on the day it is due. Style and correctness will be graded – be neat and thoroughly explain each step. Anything that is not clear will be counted wrong.

**Problem 1 (20):** For this problem use the TM $M_1$, which appears in Example 3.9 on page 173. For each part, give the sequence of configurations that $M_1$ enters when started on the input string ending with a reject or accept.

(a) 0#0
(b) 1##1
(c) 10#11
(d) 10#10

**Problem 2 (20):** Give a state transition diagram for the Turing Machine $M_3$ specified in Example 3.11 which decides the language $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$.

**Problem 3 (25):** Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet $\Sigma = \{a, b\}$.

(a) $\{w \mid w \text{ contains twice as many } a \text{'s as } b \text{'s}\}$
(b) $\{w \mid w \text{ does not contain twice as many } a \text{'s as } b \text{'s}\}$

**Problem 4 (20):** Which of the following problems about Turing machines are solvable, and which are undecidable? Explain your answers carefully.

(a) To determine, given a Turing machine $M$, a state $q$, and a string $w$, whether $M$ ever reaches state $q$ when started with input $w$ from its initial state.
(b) To determine, given a Turing machine $M$ and a symbol $a$, whether $M$ ever writes the symbol $a$ when started with the empty tape.

**Problem 5 (15):** Assume that A is reducible to B and B is reducible to C. Show that A is reducible to C.

**Bonus (2):** Favorite cult classic movie?