Molecular Computation Homework

Topic: Probability and Randomization

1 Problems

1. Suppose you are given a biased coin that has $Pr[HEADS] = p$, with $p \geq a$ for some fixed a, without being given any other information about p. Devise a procedure for estimating p by a value p' such that you can guarantee that $Pr(|p - p'| > p\epsilon) < \delta$, for any choice of the constants $0 < a$, ϵ, $\delta < 1$. Let N be the number of times you need to flip the biased coin to obtain the estimate. What is the smallest value of N for which you can still give this guarantee?

2. Suppose you are given a coin for which the probability of HEADS, say p, is unknown. How can you use this coin to generate unbiased (i.e., Pr[HEADS] =Pr[TAILS] = 1/2) coin-flips? Give a scheme for which the expected number of flips of the biased coin for extracting one unbiased coin-flip is no more than 1/(p(1 - p)). (Hint: Consider two consecutive flips of the biased coin.)