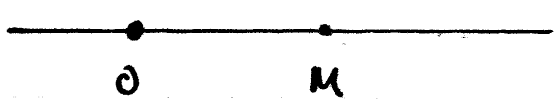
Interstellar Traveling Salesman Problem

- Some distances:

Earth - Sun	8 light-min
Earth - Neptune	4.16 light-hours
Proxima Centauri	4.3 light-years
Orion Nebula	1.500 light-years
Center of Milky Way galaxy	26.000 light-years
Andromeda Galaxy	2.36 million light-years.

- Speed of light not an obvious limitation because of <u>time-dilation</u>.

- Newtonian - mechanics:



$$x(t) = OM$$
$$u(t) = dx(t)/dt$$
$$a(t) = du(t)/dt$$

$F = ma \Rightarrow$ constant acceleration

$$x(t) = \frac{1}{2} a t^2 .$$

- Relativistic mechanics:

$$F = \frac{d}{dt}(mu)$$

$$m = \frac{m_0}{\sqrt{1-u^2}}$$

$$T = \int_0^t dt\,\sqrt{1-u^2} \leftarrow \text{Time dilation.}$$

- Assume constant force

$F = m_0 g \longleftarrow$ artificial gravity.

$$g = \frac{d}{dt}\left(\frac{u}{\sqrt{1-u^2}}\right) \Leftrightarrow \frac{u}{\sqrt{1-u^2}} = gt \Leftrightarrow$$

$$\Leftrightarrow \frac{u^2}{1-u^2} = (gt)^2 \Leftrightarrow u^2 = (gt)^2 - (gt)^2 u^2$$

$$\Leftrightarrow u^2 = \frac{(gt)^2}{1+(gt)^2} \Leftrightarrow \boxed{u = \sqrt{\frac{(gt)^2}{1+(gt)^2}}}$$

Distance traveled

$$s = \int_0^t u(\tau)\,d\tau = \int_0^t \frac{g\tau}{\sqrt{1+(g\tau)^2}}\,d\tau.$$

Let $y = 1+(g\tau)^2 \Rightarrow dy = 2g^2\tau\, d\tau$

$\tau = 0 \Rightarrow y = 1$

$\tau = t \Rightarrow y = 1+(gt)^2$ $\Rightarrow$

$$\Rightarrow s = \int_1^{1+(gt)^2} \frac{1/2\, dy}{g\sqrt{y}} = \left[\frac{\sqrt{y}}{g}\right]_1^{1+(gt)^2} =$$

$$= \left[\sqrt{1+(gt)^2} - 1\right]\cdot\frac{1}{g}$$

thus $\boxed{s(t) = \left(\sqrt{1+(gt)^2} - 1\right)\cdot\frac{1}{g}}$

Dilated Time

$$T = \int_0^t \sqrt{1-u^2}\,d\tau = \int_0^t \sqrt{1 - \frac{(g\tau)^2}{1+(g\tau)^2}}\,d\tau =$$

$$= \int_0^t \frac{d\tau}{\sqrt{1+(g\tau)^2}}$$

Let $g\tau = \tan y \Leftrightarrow y = \mathrm{Arctan}(g\tau)$

$$g\,d\tau = (1+\tan^2 y)\,dy \Rightarrow d\tau = \frac{1}{g}(1+\tan^2 y)\,dy$$

For $\tau = 0$: $y = 0$

$\tau = t$: $y = \mathrm{Arctan}(gt)$

thus

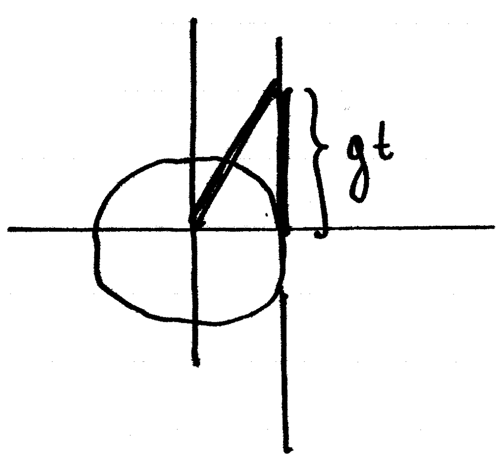
$$T = \int_0^{\mathrm{Arctan}(gt)} \frac{1}{g}\,\frac{(1+\tan^2 y)}{\sqrt{1+\tan^2 y}}\,dy =$$

$$= \int_0^{\mathrm{Arctan}(gt)} \frac{1}{g}\sqrt{1+\tan^2 y}\;dy =$$

$$= \int_0^{\mathrm{Arctan}(gt)} \frac{1}{g}\,\frac{1}{\cos y}\,dy = \cdots =$$

$$= \left[\frac{1}{g}\,\frac{1}{2}\ln\left(\frac{|\sin(y)+1|}{|\sin(y)-1|}\right)\right]_{y=0}^{y=\mathrm{Arctan}(gt).}$$

- Use $\sin(\mathrm{Arctan}(gt)) = \dfrac{gt}{\sqrt{1+(gt)^2}}$



- Challenge: Want T as a function of s!!

It follows that for $y = \mathrm{Arctan}(gt)$

$$|1+\sin y| = \frac{gt + \sqrt{1+(gt)^2}}{\sqrt{1+(gt)^2}}$$

$$|1-\sin y| = \frac{-gt + \sqrt{1+(gt)^2}}{\sqrt{1+(gt)^2}}$$

thus

$$\boxed{\tau = \frac{1}{2g} \ln\left[\frac{\sqrt{1+(gt)^2} + gt}{\sqrt{1+(gt)^2} - gt}\right]}$$

If $g_0 = 9.81\,m/sec^2$ (non-relativistic accelaration) we use

$$g = g_0/c.$$