Navier-Stokes equation with very rough data

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We study the Cauchy problem for the incompressible Navier-Stokes equation (NS):

$$\begin{aligned} u_t - \Delta u + u \cdot \nabla u + \nabla p = 0, \\ \operatorname{div} u = 0, \\ u(0,x) = u_0. \end{aligned}$$

We consider a class of very rough initial data in $E^s_{2,2}$ for which the norm are defined by

$$\|u_0\|_{E^s} = \|2^{s|\xi|}\widehat{u}_0(\xi)\|_{L^2}, \quad s < 0$$

and show that NS has a unique global solution if the initial value $u_0 \in E^s$, $s < 0$ and their Fourier transforms are supported in $\mathbb{R}^d_I := \{\xi \in \mathbb{R}^d : \ \xi_i \geq 0, \, i = 1, ..., d\}$. Our results imply that NS has a unique global solution if the initial value u_0 is in L^2 with $\operatorname{supp} \widehat{u}_0 \subset \mathbb{R}^d_I$. This is a joint work with H. Feichtinger, K. Gröchenig and Kuijie Li.