Outgoing Fredholm theory and the limiting absorption principle for asymptotically conic spaces

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In this talk I will discuss geometric generalizations of Euclidean resolvent estimates, such as estimates for the resolvent of the Laplacian of an asymptotically conic metric plus a decaying potential, in a Fredholm framework that focuses on capturing the outgoing asymptotics of the resolvent applied to a Schwartz function (outgoing waves); this is different from even the usual treatment of the Euclidean problem. More precisely, the setting is that of perturbations $P(\sigma)$ of the spectral family of the Laplacian $\Delta_g - \sigma^2$ on asymptotically conic spaces (X, g) of dimension at least 3 (with the asymptotic behavior at the 'large end' of the cone), and the main results are the limiting absorption principle, as well as uniform estimates for $P(\sigma)^{-1}$ as $\sigma \to 0$, on function spaces between which $P(\sigma)$ is Fredholm even for real $\sigma \neq 0$ and which correspond to finite regularity Lagrangian distributions associated to the, conic in the base, Lagrangian given by the outgoing radial set of the Hamilton flow. Such results have immediate applications to the behavior of the wave equation on black hole spacetimes.